Coasting Arcs in Optimal Power-Limited Rocket Flight

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One of the problems treated in Ref. 1 is that of optimal thrust control for power-limited flight. It is shown there that operation at maximum propulsive power is optimal, provided limitation is on power alone. In effect, it can be shown that operation at maximum power is optimal, provided the thrust is nonzero, but that coasting flight may be optimal under special, highly restricted, circumstances. In the ensuing analysis, the nomenclature of Ref. 1 is used.

The optimal thrust-magnitude program may be found from the maximum principle—namely, power α and mass-flow rate β must be such as to maximize

$$L = (\sqrt{\alpha\beta/m})\sqrt{\lambda_u^2 + \lambda_v^2} - \lambda_m\beta \tag{1}$$

where

$$0 \leq \alpha \leq \alpha_{max} \tag{2}$$

and β is not assumed bounded. With respect to α, provided $\beta \neq 0$, it is required that

$$\alpha = \alpha_{max} \tag{3}$$

With respect to β, two cases may arise, depending on the adjoint variable λ_m, associated with mass variation. These are illustrated in Fig. 1.

Thus, if

$$\lambda_m \leq 0 \tag{4}$$

L does not possess a stationary maximum and the optimal choice of β is the largest possible value. If

$$\lambda_m > 0 \tag{5}$$

there always exists a stationary maximum of L, and the optimal choice of β is given by

$$\partial L/\partial\beta = 0 \tag{6}$$

A solution of Eq. (6) corresponding to coasting flight—i.e.,

$$\beta \equiv 0 \tag{7}$$

can occur only if

$$\lambda_u = \lambda_v \equiv 0 \tag{8}$$

and the latter condition implies

$$\lambda_x = \lambda_y \equiv 0 \tag{9}$$

$$\lambda_m = \text{constant} \tag{10}$$

as can be seen by inspection of the adjoint equations, Eq. (5.45) of Ref. 1.

Since the adjoint variables are the partial derivatives of the payoff with respect to the state variables—i.e.,

$$\lambda_q = -\partial J/\partial q \tag{11}$$

where λ_q is the adjoint variable associated with state variable q, and J is the value of the quantity to be minimized (from state q to the end state)—it can be seen that coasting flight is optimal only if the payoff is insensitive to first-order changes in position and velocity. It is doubtful that many meaningful problems fall into this category. However, to illustrate the possibility, consider the following trivial problem.

It is desired to transfer maximum mass in a field-free environment between equal velocities without restriction on position—i.e.,

$$\min(-m_f) \tag{12}$$

with end conditions

$$\left.\begin{array}{l} t = 0:\ m = m_0,\quad x = y = 0,\quad u = u_0,\quad v = 0 \\ t = t_f:\ u = u_0,\quad v = 0 \end{array}\right\} \tag{13}$$

Of course, the answer is $m_f = m_0$ and the trajectory is solely coasting. Conditions (8) to (10) are clearly met. Since for field-free flight

$$\lambda_x = \text{const.},\quad \lambda_y = \text{const.} \tag{14}$$

and by Eq. (11)

$$\lambda_x = \lambda_y = 0 \tag{15}$$

and also

$$\lambda_u = \lambda_v = 0 \tag{16}$$

Furthermore, by Eq. (11)

$$\lambda_m = -\partial(-m_f)/\partial m_0 = 1 > 0 \tag{17}$$

so that all conditions are met.

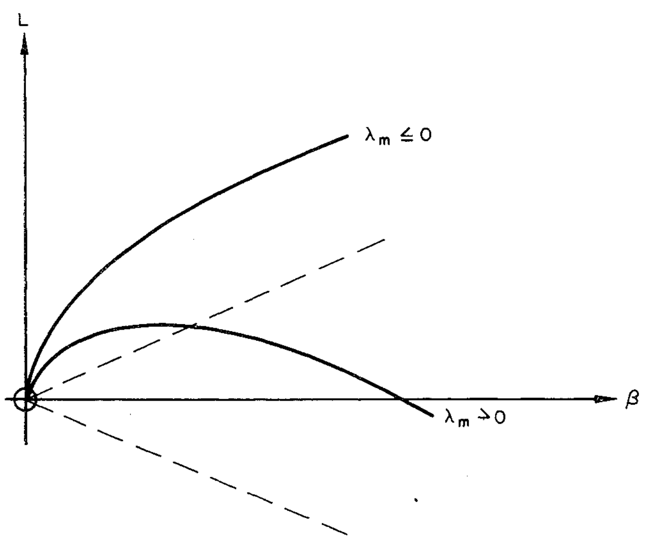

Fig. 1.

Reference

[1] Leitmann, G., *Optimization Techniques*, Chap. 5, pp. 182f; Academic Press, New York, 1962.

A Matrix Formulation of the Transverse Structural Influence Coefficients of an Axially Loaded Timoshenko Beam

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A set of structural influence coefficients (SICs) is required to carry out a vibration analysis by collocation methods. For applications to many missile configurations, the structure often may be assumed to be statically determinate and idealized as a Timoshenko beam—i.e., both bending and shearing deformations are considered. In addition, the effect of axial thrust must be considered, for, although the thrust is usually much less than the buckling load, the small changes in the transverse-vibration frequencies due to thrust are important in the design of filters in the guidance-and-control system. A routine formulation of the SICs may be obtained from the methods of elastic energy. We follow the method of Ogness.[1]

Consider a statically determinate system composed of N structural elements. The total strain energy in bending and shearing deformations is given by